



# Vacuum Insertion Approximation and the $\Delta I = 1/2$ rule: A lattice QCD test of the naïve factorization hypothesis for $K$ , $D$ , $B$ and static mesons



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## ARTICLE INFO

### Article history:

Received 30 January 2014

Received in revised form 19 May 2014

Accepted 9 July 2014

Available online 16 July 2014

Editor: G.F. Giudice

## ABSTRACT

Motivated by a recent paper by the RBC-UKQCD Collaboration, which observes large violations of the naïve factorization hypothesis in  $K \rightarrow \pi\pi$  decays, we study in this paper the accuracy of the Vacuum Insertion Approximation (VIA) for the matrix elements of the complete basis of four-fermion  $\Delta F = 2$  operators. We perform a comparison between the matrix elements in QCD, evaluated on the lattice, and the VIA predictions. We also investigate the dependence on the external meson masses by computing matrix elements for  $K$ ,  $D_s$ ,  $B_s$  and static mesons. In commonly used renormalization schemes, we find large violations of the VIA in particular for one of the two relevant Wick contractions in the kaon sector. These deviations, however, decrease significantly as the meson mass increases and the VIA predictions turn out to be rather well verified for  $B$ -meson matrix elements and, even better, in the infinite mass limit.

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## 1. Introduction

A recent paper [1] by the RBC-UKQCD Collaboration provides an emerging explanation for the “ $\Delta I = 1/2$  rule” in  $K \rightarrow \pi\pi$  decays. This rule refers to the empirical observation that the real part  $\text{Re}(A_0)$  of the amplitude describing the kaon decay in two pions with total isospin  $I = 0$  is larger by approximately a factor 22.5 than the corresponding amplitude  $\text{Re}(A_2)$  of the  $I = 2$  channel. Perturbative QCD evolution of current–current operators from the electroweak scale down to about 1.5–2 GeV contributes a factor of approximately 2 to the ratio  $\text{Re}(A_0)/\text{Re}(A_2)$  [2,3]. Therefore, barring significant new physics contributions to the decay amplitudes, the remaining factor of about 10 should come from non-perturbative QCD.

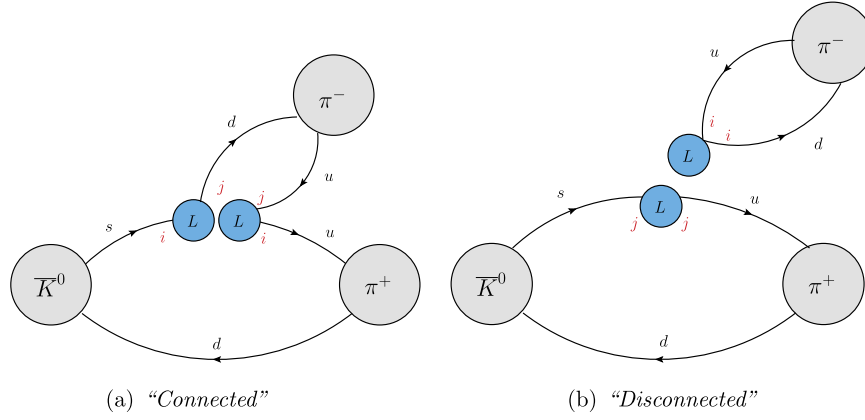
The explanation of the  $\Delta I = 1/2$  rule which is emerging from the lattice QCD studies [1,4–7] is that the two dominant contributions to the  $\Delta I = 3/2$ ,  $K \rightarrow \pi\pi$  correlation functions, which are shown diagrammatically in Fig. 1, have opposite signs leading to a significant cancellation. The same contributions are also the largest ones in  $\text{Re}(A_0)$ , but now they have the same sign and so enhance this amplitude. QCD and electroweak penguins operators, which only enter the  $\Delta I = 1/2$  transition, make only very small contributions.

While the calculation of  $\text{Re}(A_2)$  by the RBC-UKQCD Collaboration is completed, the calculation of  $\text{Re}(A_0)$  has been only performed at unphysical kinematics, with pion masses of about 330 MeV and 420 MeV. Therefore the results are not conclusive yet, and the enhancement factor of 22.5 has still to be quantitatively reproduced. Nevertheless, the emerging explanation of the  $\Delta I = 1/2$  rule discussed above is rather convincing and deserves to be further investigated.

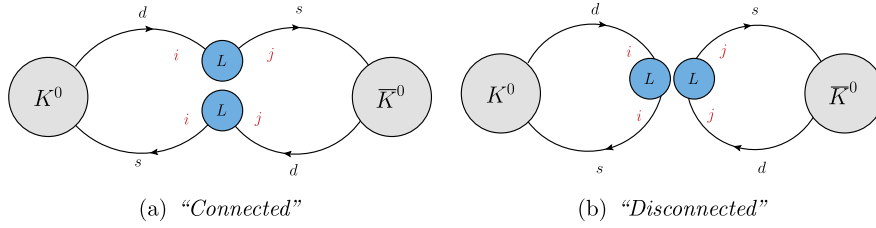
A striking feature of the lattice results for the current–current  $K \rightarrow \pi\pi$  correlators is that they almost maximally contradict the expectations of the naïve factorization hypothesis, i.e. the predictions of the Vacuum Insertion Approximation (VIA). Color counting and the VIA suggest that the connected contribution of Fig. 1 should be approximately 1/3 of the disconnected one, whereas it is found that in QCD they have opposite signs. As recently stressed in Ref. [8], this result was already obtained almost thirty years ago by Bardeen, Buras and Gérard using a model based on the dual representation of QCD as a theory of weakly interacting mesons for large  $N$  [9].

It is tempting to establish a connection between the validity of naïve factorization for emission topologies and the  $\Delta I = 1/2$  rule. In the  $K$  system, one observes a maximal deviation from the VIA and a large suppression of the  $\Delta I = 3/2$  amplitude. In non-leptonic charm decays, early analyses found moderate violations of naïve factorization which could be described by setting to zero

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**Fig. 1.** “Connected” and “disconnected” contributions to  $K \rightarrow \pi\pi$  decays in the local  $V - A$  theory. The blue circles indicate the insertion of the current–current operator with left chirality. The two diagrams are distinguished by the summation of the spin (single and double trace) and color ( $i, j$ ) indices.



**Fig. 2.** “Connected” and “disconnected” contributions to  $\bar{K}^0 - K^0$  mixing. The notation is the same as in Fig. 1.

the  $1/N_c$ -suppressed terms in the factorized matrix elements [10]. Correspondingly, one observes comparable  $\Delta I = 1/2$  and  $\Delta I = 3/2$  amplitudes, with a large relative phase [11]. Finally, in the  $B$  system factorization for emission topologies has been demonstrated in the infinite mass limit [12], and  $B \rightarrow \pi\pi$  decays can be theoretically well described by factorization once the dominant sub-leading corrections are taken into account [13].

As already noted in Ref. [1], a violation of similar extent of the VIA is also exhibited by the connected and disconnected contributions to the matrix element  $\langle \bar{K}^0 | (\bar{s}\gamma_L^\mu d)(\bar{s}\gamma_L^\mu d) | K^0 \rangle$  which contains the non-perturbative QCD effects in neutral kaon mixing (see Refs. [8,14] for a detailed discussion of this matrix element in the context of large  $N$ ). By using  $SU(3)$  flavor symmetry it can be shown, in fact, that the matrix elements of the  $\Delta S = 2$  operator for  $\bar{K}^0 - K^0$  mixing and of the  $\Delta I = 3/2$  operator for  $K \rightarrow \pi\pi$  decays are proportional in the soft pion limit [15]. For this reason, earlier attempts to study  $K \rightarrow \pi\pi$  decays on the lattice were based on the evaluation of the matrix element of the  $\Delta I = 3/2$  operator between a kaon and a single pion state.

The connected and disconnected diagrams contributing to the  $\langle \bar{K}^0 | (\bar{s}\gamma_L^\mu d)(\bar{s}\gamma_L^\mu d) | K^0 \rangle$  matrix element are shown in Fig. 2. They originate from the same Wick contractions as the analogous diagrams for the  $K \rightarrow \pi\pi$  matrix element presented in Fig. 1. As for the latter, the VIA predicts that also in the  $\bar{K}^0 - K^0$  case the two contributions come in the ratio of  $1/3 : 1$ , whereas the lattice calculations show that in QCD they have opposite signs.

In this letter, we further extend the comparison between QCD and VIA predictions for the four-fermion operator matrix elements in several respects. In particular: i) we confirm the findings of Ref. [1] for the connected and disconnected contributions to the matrix element of the left–left current operator between external kaon states; ii) we extend the comparison between QCD and VIA predictions to the whole 10-dimensional basis of four-fermion operators, characterized by different spin and color structures; iii) we extend the comparison to the matrix elements of heavier mesons than the kaons, namely  $D$  and  $B$  mesons as well as *static* mesons,

i.e. mesons constituted by a heavy quark of infinite mass. Our main results show that the VIA predictions are largely violated in QCD also for other operators besides the left–left current operators, particularly for the connected contributions. The discrepancies, however, decrease significantly as the meson mass increases, and the VIA predictions turn out to be rather well verified for  $B$ -meson matrix elements and, even better, in the infinite mass limit. Although numerical results are presented in  $\overline{MS}$  scheme at 3 GeV, the above qualitative conclusions do not depend, to a large extent, on the renormalization scheme and scale.

Our numerical results have been obtained by using the gauge configurations generated by European Twisted Mass Collaboration (ETMC) with  $N_f = 2$  dynamical quarks at four values of the lattice spacing [16]. The 2- and 3-point correlation functions analyzed for the present study have been computed to evaluate the matrix elements of the four-fermion operators relevant for  $\bar{K}^0 - K^0$ ,  $\bar{D}^0 - D^0$  and  $\bar{B}^0 - B^0$  mixing, within and beyond the Standard Model, in Refs. [17,18], [19] and [20] respectively. The results for the matrix elements between external  $B$  and static meson states have been obtained by implementing the so-called ratio method for heavy quarks developed in Refs. [21,22] and optimized smearing techniques [20].

## 2. Matrix elements in the VIA

In order to study separately the connected and disconnected contributions to the matrix element of the  $\Delta F = 2$  four-fermion operator, shown in Fig. 2 for the kaon case, we find convenient to consider the following  $\Delta F = 1$  operators

$$\begin{aligned} O_{\Gamma\Gamma} &= (\bar{h}\Gamma\ell)(\bar{h}'\Gamma\ell') \\ O_{\Gamma\Gamma}^F &= (\bar{h}\Gamma\ell')(\bar{h}'\Gamma\ell), \end{aligned} \quad (1)$$

where  $h, h', \ell, \ell'$  are different quark fields and  $\Gamma$  is a generic Dirac matrix. It is easy to realize that the matrix elements of the operators  $O_{\Gamma\Gamma}$  and  $O_{\Gamma\Gamma}^F$  between external mesons of flavor content

**Table 1**

Details of the lattice simulations used for the present study. We provide the approximate value of the lattice spacing ( $a$ ) [25], the number of lattice sites in the spatial ( $L$ ) and temporal ( $T$ ) directions, the number ( $N_{\text{stat}}$ ) of independent gauge configurations for each ensemble, the values of the bare quark masses in lattice units in the light ( $\mu_\ell$ ), strange ( $\mu_s$ ) and heavy ( $\mu_h$ ) quark mass regions.

$a$ (fm)	$L^3 \times T$	$N_{\text{stat}}$	$a\mu_\ell = a\mu_{\text{sea}}$	$a\mu_s$	$a\mu_h$
0.098	$24^3 \times 48$	128	0.0080 0.0110	0.0175	0.1982 0.2331 0.2742
				0.0194	0.3225 0.3793 0.4461
				0.0213	0.5246 0.6170
0.085	$24^3 \times 48$	240	0.0040 0.0064	0.0159	0.1828 0.2150 0.2529
				0.0177	0.2974 0.3498 0.4114
	$32^3 \times 64$	144	0.0030 0.0040	0.0195	0.4839 0.5691
0.067	$32^3 \times 64$	144	0.0030 0.0060 0.0080	0.0139	0.1572 0.1849 0.2175
				0.0154	0.2558 0.3008 0.3538
				0.0169	0.4162 0.4895
0.054	$32^3 \times 64$	144	0.0065	0.0116	0.13315 0.1566 0.1842
				0.0129	0.2166 0.2548 0.2997
	$48^3 \times 96$	80	0.0020	0.0142	0.3525 0.4145

$(\bar{h}\ell)$  and  $(\bar{h}'\ell')$  receive contribution only from the disconnected and connected contraction respectively. In this paper, the flavors  $h$  and  $h'$  are always taken to be degenerate in mass, and similarly for  $\ell$  and  $\ell'$ . We will then present results for  $(h, \ell) = (s, d)$ ,  $(c, s)$ , and  $(b, s)$  for kaon,  $D_s$  and  $B_s$  mesons respectively. Moreover, we will also extrapolate the heavy quark mass to the infinite quark mass in order to investigate the accuracy of the VIA approximation in the static limit.

For the Dirac structure of the four-fermion operators we consider the following (complete) basis of operators:

$$O_X^{(F)} = \{O_{VV+AA}^{(F)}, O_{VV-AA}^{(F)}, O_{SS-PP}^{(F)}, O_{SS+PP}^{(F)}, O_{SS+PP-TT/2}^{(F)}\} \quad (2)$$

where  $O_{VV\pm AA}^{(F)} = O_{VV}^{(F)} \pm O_{AA}^{(F)}, \dots$  and  $V, A, S, P, T$  stand for  $\gamma^\mu, \gamma^\mu\gamma^5, 1, \gamma^5, \sigma^{\mu\nu}$ . With this choice, all operators have non-vanishing matrix elements in the VIA, which read:

$$\begin{aligned} \langle P^0 | O_{VV+AA} | \bar{P}'^0 \rangle_{\text{VIA}} &= -\langle P^0 | O_{VV-AA} | \bar{P}'^0 \rangle_{\text{VIA}} = F^2 M^2 \\ \langle P^0 | O_{SS-PP} | \bar{P}'^0 \rangle_{\text{VIA}} &= -\langle P^0 | O_{SS+PP} | \bar{P}'^0 \rangle_{\text{VIA}} = -\langle P^0 | O_{SS+PP-TT/2} | \bar{P}'^0 \rangle_{\text{VIA}} \\ &= \frac{F^2 M^4}{(m_h + m_\ell)^2}, \end{aligned} \quad (3)$$

where  $M$  and  $F$  are the mass and decay constants of the pseudoscalar  $(\bar{h}\ell)$ -meson  $P^0$  and  $m_{h(\ell)}$  the corresponding quark masses. As is well known, both the scheme and scale dependence of operators and quark masses in the VIA is neglected.

The matrix elements of the  $O^F$  operators in the VIA are obtained after a Fierz transformation of both spin and color indices:

$$\langle P^0 | O_X^F | \bar{P}'^0 \rangle = \frac{1}{3} F_{XY} \langle P^0 | O_Y | \bar{P}'^0 \rangle + \frac{1}{2} F_{XY} \langle P^0 | O_Y^\lambda | \bar{P}'^0 \rangle, \quad (4)$$

where  $F_{ij}$  is the Dirac Fierz matrix

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & -2 & 0 \end{pmatrix} \quad (5)$$

and the operators  $O_X^\lambda$  are defined as

$$O_{\Gamma\Gamma}^\lambda = (\bar{h}\lambda^a \Gamma \ell)(\bar{h}'\lambda^a \Gamma \ell') \quad (6)$$

with  $\lambda^a$  the color group generators (Gell-Mann matrices). In the VIA, one simply has:

$$\langle P^0 | O_X^\lambda | \bar{P}'^0 \rangle_{\text{VIA}} = 0, \quad (7)$$

and thus, from Eq. (4)

$$\begin{aligned} \langle P^0 | O_{VV+AA}^F | \bar{P}'^0 \rangle_{\text{VIA}} &= \frac{1}{3} F^2 M^2 \\ \langle P^0 | O_{VV-AA}^F | \bar{P}'^0 \rangle_{\text{VIA}} &= -\frac{2}{3} \frac{F^2 M^4}{(m_h + m_\ell)^2} \\ \langle P^0 | O_{SS-PP}^F | \bar{P}'^0 \rangle_{\text{VIA}} &= \frac{1}{6} F^2 M^2 \\ \langle P^0 | O_{SS+PP}^F | \bar{P}'^0 \rangle_{\text{VIA}} &= \frac{1}{6} \frac{F^2 M^4}{(m_h + m_\ell)^2} \\ \langle P^0 | O_{SS+PP-TT/2}^F | \bar{P}'^0 \rangle_{\text{VIA}} &= \frac{2}{3} \frac{F^2 M^4}{(m_h + m_\ell)^2}. \end{aligned} \quad (8)$$

In order to investigate the accuracy of the VIA, in the following we will present the results in terms of ratios between the matrix elements in QCD and their expression in the VIA,

$$R_X^{(F)} = \frac{\langle P^0 | O_X^{(F)} | \bar{P}'^0 \rangle}{\langle P^0 | O_X^{(F)} | \bar{P}'^0 \rangle_{\text{VIA}}}. \quad (9)$$

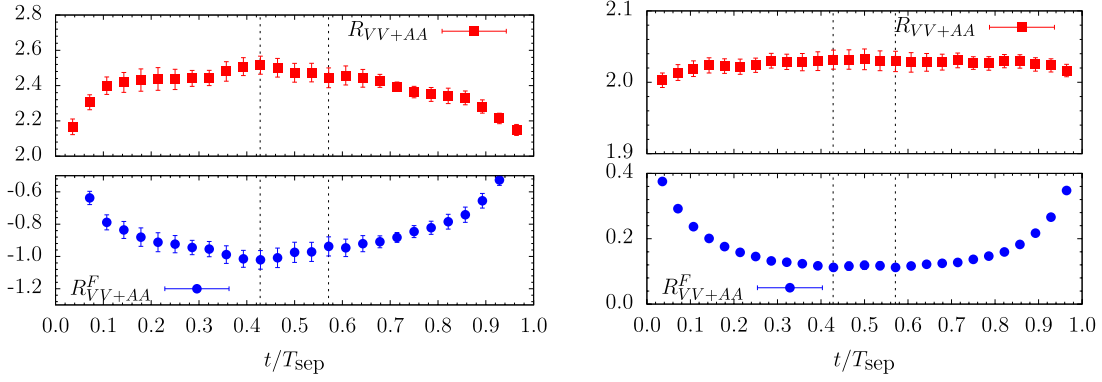
For the matrix elements of the operators  $O_X^\lambda$ , which vanish in the VIA, we adopt the following normalization:

$$R_X^\lambda = \frac{\langle P^0 | O_X^\lambda | \bar{P}'^0 \rangle}{\langle P^0 | O_X | \bar{P}'^0 \rangle_{\text{VIA}}}, \quad (10)$$

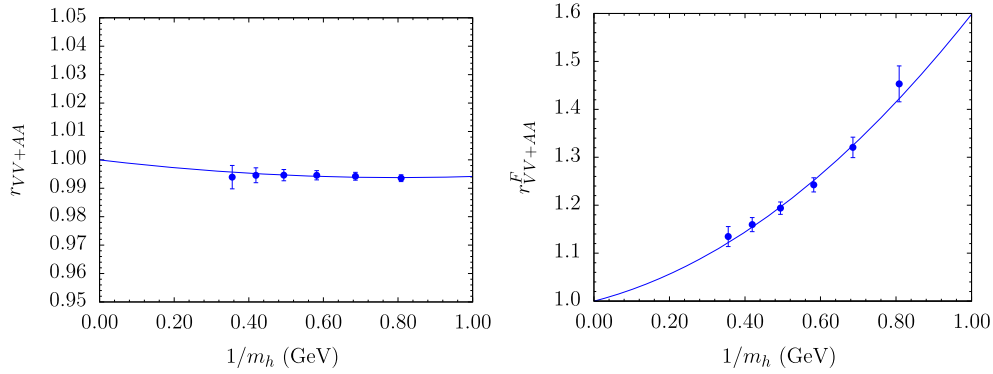
and compute the matrix elements of  $O_X^\lambda$  using Eq. (4).

### 3. Results

The lattice calculation of the matrix elements in QCD has been performed at four values of the lattice spacing, using the  $N_f = 2$  dynamical quark configurations produced by the ETM Collaboration [16,23]. Quark fields are regularized by employing the twisted mass/Osterwalder–Seiler formalism at maximal twist, which guarantees automatic  $\mathcal{O}(a)$ -improvement and continuum-like renormalization pattern for the four-fermion operators [24]. In Table 1 we provide the main simulation details, including the values of (bare) quark masses for each lattice spacing. The values of the light (up and down) quark masses are equal for sea and valence quarks.



**Fig. 3.** Lattice data and time plateau for the estimators of  $R_{VV+AA}^{(F)}$  as a function of  $t/T_{\text{sep}}$ , where  $t$  is the Euclidean time and  $T_{\text{sep}}$  is the separation between the two external pseudoscalar meson sources. We show results for the finest lattice spacing ( $a \simeq 0.054$  fm) and the lightest quark mass ( $a\mu_\ell = 0.0020$ ). Left and right panels correspond to the light-strange ( $K$ ) and strange-charm ( $D_s$ ) pseudoscalar mesons respectively. The dotted lines delimit the plateau region from which the results for the ratios  $R_{VV+AA}^{(F)}$  are extracted.



**Fig. 4.** Results for the ratios  $r_{VV+AA}$  (left) and  $r_{VV+AA}^{(F)}$  (right) as a function of  $1/m_h$ , where  $m_h$  is the heavy quark mass renormalized in the  $\overline{\text{MS}}$  scheme at the  $\mu = 3$  GeV. The solid lines illustrate the result of a quadratic fit of the lattice data and the precisely known value  $r_X^{(F)} = 1$  in the infinite mass limit.

We then simulate three valence quark masses around the physical strange mass and a set of heavy valence quark masses in the range between  $m_c$  and  $2.5m_c$ , where  $m_c$  is the physical charm mass.

The lattice computation of the relevant four-fermion matrix elements proceeds as discussed in Refs. [17,18], [19] and [20] for  $\bar{K}^0-K^0$ ,  $\bar{D}^0-D^0$  and  $\bar{B}^0-B^0$  mixing respectively. In the present study, however, we evaluate separately the contributions of the connected and disconnected diagrams and compare them with the VIA predictions. In Fig. 3 we show, as an example, the lattice estimators for the ratios  $R_{VV+AA}$  and  $R_{VV+AA}^{(F)}$  as a function of the Euclidean time in the kaon and  $D$ -meson case. The values of the ratios are extracted from the central region in which the time dependent correlators exhibit a plateau.

The renormalization constants of the two- and four-fermion operators have been computed non-perturbatively in the RI-MOM scheme in [18,26] and converted to  $\overline{\text{MS}}$  using continuum perturbation theory. For renormalizing the operators in the basis of Eq. (2), we simply performed a change of basis from the four-fermion renormalization matrix reported in [18].

For the calculation of  $B$  and static meson matrix elements we have applied the ratio method for heavy quarks and optimized smearing techniques, as discussed in Ref. [20]. The method relies on the construction of suitable ratios with exactly known static limit and an interpolation between the lattice results evaluated in the accessible charm region and the infinite mass point. In the case of interest, the quantities  $R_X^{(F)}$  defined in Eq. (9) tends to a constant in the infinite mass limit. Therefore, double ratios as

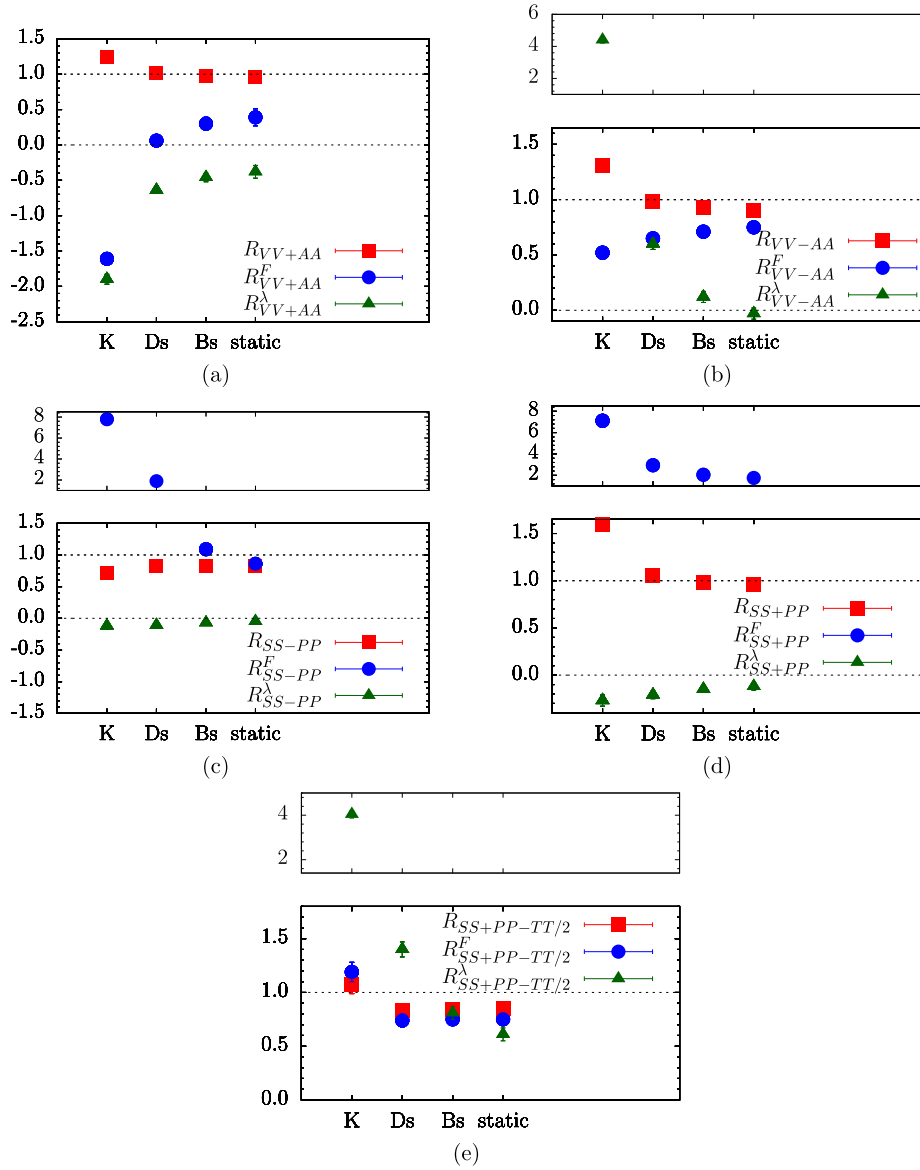
$$r_X^{(F)}(m_h) = \frac{R_X^{(F)}(m_h)}{R_X^{(F)}(m_h/\lambda)} \quad (11)$$

are equal to 1 in the static limit up to logarithmic corrections which can be evaluated in perturbation theory. In practice, with this method, the results at the  $b$ -quark mass are obtained after a relatively small, typically  $N_s = 9$ , number of steps (the precise value of  $N_s$  depends on  $\lambda$ ). By iterating the same procedure for a much larger value of steps,  $N_s = \mathcal{O}(40)$ , one reaches numerically an asymptotic result which corresponds to the static limit. Two examples of the lattice data for the double ratios in Eq. (11), namely  $r_{VV+AA}$  and  $r_{VV+AA}^{(F)}$ , and of their interpolation to heavier quark masses are shown in Fig. 4.

Our final results in the continuum limit for the ratios between QCD and VIA matrix elements in the  $K$ ,  $D_s$ ,  $B_s$  and static meson sectors are collected in Table 2 and shown in Fig. 5. The four-fermion operators are renormalized in the  $\overline{\text{MS}}$  scheme of Ref. [27] at the scale  $\mu = 3$  GeV. As a cross check of the calculation, we verified that, by properly combining the results for the connected and disconnected matrix elements given in Table 2, we are able to reproduce the results for the bag parameters  $B_i$  obtained in Refs. [18–20].

#### 4. Conclusions

The results collected in Table 2 and presented in Fig. 5 show large violations of the VIA, particularly for the connected contributions in the kaon sector. The ratios  $R_{SS+PP}^F$  and  $R_{SS-PP}^F$  in this sector are found to be as large as 7–8, while the ratio  $R_{VV+AA}^F$



**Fig. 5.** Comparison among the values of the ratios  $R_X^{(F,\lambda)}$  in the  $K$ ,  $D_s$ ,  $B_s$  and static meson sectors. The results are renormalized in the  $\overline{\text{MS}}$  scheme of Ref. [27] at the scale  $\mu = 3$  GeV. For clarity, the y-axis in figures (b), (c), (d) and (e) have been split.

has negative sign, as anticipated in Ref. [1,4]. The deviations from the VIA decrease significantly, however, as the meson mass increases. The ratio  $R_{SS-PP}^F$ , for instance, becomes compatible with 1 at the  $B$  meson mass region, and the ratio  $R_{VV+AA}^F$  changes its sign around the charm mass region. Large deviations from the VIA in the kaon sector are observed also for the matrix elements of the four-fermion operators with the octet color structure, and some of the ratios  $R_X^\lambda$ , which are expected to vanish in the VIA, are found to be much larger than 1. As in the case of the color singlet operators, these deviations become significantly smaller going towards the static limit.

Results similar to those presented in Table 2 and Fig. 5 are obtained when the operators are renormalized in the  $\overline{\text{MS}}$  scheme at the scale of 2 GeV or in the RI-MOM scheme at the same scales. The scheme dependence is an  $\mathcal{O}(\alpha_s)$  effect and all our conclusions about the accuracy of the VIA remain qualitatively valid. The numerical results in Table 2 and Fig. 5 are obtained in the  $N_f = 2$  theory and do not account for the dynamical sea quark effects of the strange and heavier quarks. However, preliminary

ETMC results with  $N_f = 2 + 1 + 1$  dynamical sea quarks indicates that the systematic effect due to the partial quenching do not change the qualitative conclusions described here. We also notice that once the results for the ratios  $R_X$  and  $R_X^F$  are combined in order to reconstruct the values of the B-parameters for the  $\Delta F = 2$  operators, the large violations of the VIA observed particularly in the kaon case cancel to a large extent. The B-parameters for the five independent operators turn out to be of order one (see Refs. [18–20]).

## Acknowledgements

We warmly thank Petros Dimopoulos and Cecilia Tarantino for useful discussions and for comments on the manuscript. The research leading to these results has received funding from the MIUR (Italy) under the contract PRIN 2010–2011 and from the European Research Council under the European Union’s Seventh Framework Programme (FP/2007–2013)/ERC Grant Agreements No. 279972 and No. 267985.

**Table 2**

Results for the ratios  $R_X^{(F,\lambda)}$  between the matrix elements of the four-fermion operators in QCD and in the VIA, defined in Eqs. (9) and (10). The ratios are renormalized in the  $\overline{\text{MS}}$  scheme of Ref. [27] at the scale  $\mu = 3$  GeV.

	$K$	$D_s$	$B_s$	Static limit
$R_{VV+AA}$	1.24(04)	1.02(03)	0.98(03)	0.96(04)
$R_{VV-AA}$	1.31(05)	0.98(03)	0.93(03)	0.90(03)
$R_{SS-PP}$	0.71(03)	0.82(03)	0.82(04)	0.83(04)
$R_{SS+PP}$	1.60(05)	1.06(04)	0.98(04)	0.96(03)
$R_{SS+PP-TT/2}$	1.07(08)	0.83(05)	0.84(05)	0.85(05)
$R_{VV+AA}^F$	-1.61(08)	0.06(02)	0.30(09)	0.39(12)
$R_{VV-AA}^F$	0.52(04)	0.65(04)	0.71(05)	0.74(05)
$R_{SS-PP}^F$	7.8(4)	1.89(08)	1.09(06)	0.86(06)
$R_{SS+PP}^F$	7.1(2)	2.94(11)	2.05(09)	1.75(10)
$R_{SS+PP-TT/2}^F$	1.19(09)	0.74(06)	0.75(05)	0.78(05)
$R_{VV+AA}^\lambda$	-1.90(07)	-0.64(02)	-0.46(06)	-0.38(09)
$R_{VV-AA}^\lambda$	4.3(2)	0.60(05)	0.12(05)	-0.03(05)
$R_{SS-PP}^\lambda$	-0.13(03)	-0.11(03)	-0.07(03)	-0.05(03)
$R_{SS+PP}^\lambda$	-0.27(06)	-0.21(04)	-0.15(03)	-0.12(04)
$R_{SS+PP-TT/2}^\lambda$	4.04(16)	1.40(07)	0.81(06)	0.61(06)

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